BROWN UNIVERSITY
PROBLEM SET 7
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DUE: 3 NOVEMBER 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

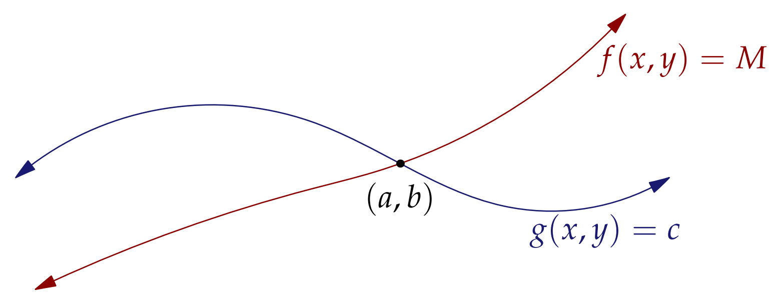
In this problem, we will provide an explanation for the Lagrange multipliers formula which has an accompanying visualization different from the one provided in the book. Your explanations do not need to be rigorous; you may assume all curves are smooth and you can use facts which are merely visually apparent from your figures.

Suppose that f and g are differentiable functions of two variables and $c \in \mathbb{R}$. Suppose that the restriction of f to the c-level set of g has a local maximum of M at (a, b), and that $\nabla g(a, b) \neq \mathbf{0}$.

(a) Suppose that the M-level set of f crosses through the c-level set of g as shown. Why does this imply that $\nabla f(a, b) = 0$? Hint: suppose that $\nabla f(a, b) \neq \mathbf{0}$ and use the fact that f is larger on one side of $f(x, y) = M$ than the other to arrive at a contradiction.

(b) Use (a) to conclude that if $\nabla f \neq \mathbf{0}$, then ∇f and ∇g point in the same or opposite directions. Draw a sketch of what the intersection of the level curves $\{(x, y) \in \mathbb{R}^2 : f(x, y) = M\}$ and $\{(x, y) \in \mathbb{R}^2 : g(x, y) = c\}$ would look like in this case.

(c) Use your figure from (b) to explain the following *duality* result: assuming $\nabla f \neq \mathbf{0}$, if f restricted to the c-level set of g has a local maximum of M, then g restricted to the M-level set of f has a local minimum or maximum of c.



Problem 2

Find the largest and smallest values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

Solution

Problem 3

Use the method of Lagrange multipliers to find the minimum distance from the origin to the plane $3x + 2y + z = 6$.

Solution

Final answer:

Problem 4

Integrate $(x + y)^2$ over the triangular region with vertices at the origin, $(3, 0)$, and $(0, 4)$.

Note: the algebra is a little tedious; click here (`http://tinyurl.com/y9zubnaa`) for help with that.

Solution

Final answer:

Problem 5

Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} \, dx \, dy$ by reversing the order of integration.

Solution

Problem 6

Set up iterated integrals for both orders of integration for the integral of $f(x, y) = y$ over the region D bounded by $y = x - 2$, $x = y^2$. Then evaluate the double integral using the easier order and explain why it's easier.

Solution

Additional space