Brown University
Problem Set 5
Instructor: Samuel S. Watson
Due: 13 October 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

(a) Find the quadratic Maclaurin polynomial for the function $f(x,y) = e^{3x+y}$ by calculating all the relevant partial derivatives.

(b) Find the quadratic Maclaurin polynomial for e^t and substitute $t = 3x + y$.

Solution

Problem 2

Consider the function $f(x,y) = \frac{e^{xy}}{e(1+x^2)}$.

(a) Use a quadratic Taylor polynomial centered at $(1,1)$ to approximate $f(0.99, 0.98)$. Compare your answer to Example 4.3.3 in the book.

(b) Use a quadratic Taylor polynomial centered at $(0,0)$ to approximate $f(0.99, 0.98)$.

(c) The following code can be copy-pasted at `sagecell.sagemath.org` (or click here) to calculate the degree-50 Maclaurin polynomial of f and evaluate it at $(0.99, 0.98)$.

```
var("x y") # declares x and y to be symbolic variables
f(x,y) = exp(x*y-1)/(1+x^2) # defines f
taylor(f(x,y),(x,0),(y,0),50).subs(x=0.99,y=0.98).n()
```

How good is this estimate compared to the ones in (a) above and in Example 4.3.3 in the text?

(d) Repeat (c) but with the function $g(x,y) = \frac{e^{xy}}{e(9+x^2)}$. Does the degree-50 Maclaurin polynomial approximate the value of $g(0.99, 0.98)$ well?

Solution

Problem 3

Consider the function $f(x, y) = \frac{1}{xy}$. Show that f has no maximum or minimum value on the open unit square $S = (0, 1) \times (0, 1)$. In other words, show that for any $(x, y) \in S$, there exists $(x', y') \in S$ with $f(x', y') > f(x, y)$ (and similarly for the minimum).

Solution

Problem 4

Let D be the closed unit disk $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. Come up with a function $f : D \to \mathbb{R}$ with the property that f does not have a maximum value on D. Explain why your function does indeed have this property.

Solution

Problem 5

Find the maximum and minimum values of $f(x, y) = x^4 + y^4 - 4xy$ on the rectangle $[0, 3] \times [0, 2]$.

Solution

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