Brown University
Problem Set 4
Instructor: Samuel S. Watson
Due: 6 October 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Find the limit, if it exists, or show that the limit does not exist, for each of the following functions:

(a) $\lim_{(x,y)\to(3,2)} (x^2y^3 - 4y^2)$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$

(c) $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4}$

Solution

Problem 2

Which of the following are true? Explain carefully.

I. If $f(x, y)$ is not defined at the point $(7, 5)$, then the limit as $(x, y) \to (7, 5)$ of $f(x, y)$ does not exist.

II. If $f(1.99, 3.01) = 105$ and f is continuous, then $\lim_{(x,y)\to(2,3)} f(x, y)$ must be at least 100.

III. If $\lim_{(x,y)\to(0,0)} f(x, y) = 0 = f(0, 0)$, then f is continuous at the origin.

Solution

Problem 3

A function $u(x, y)$ is said to satisfy *Laplace's equation* if

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

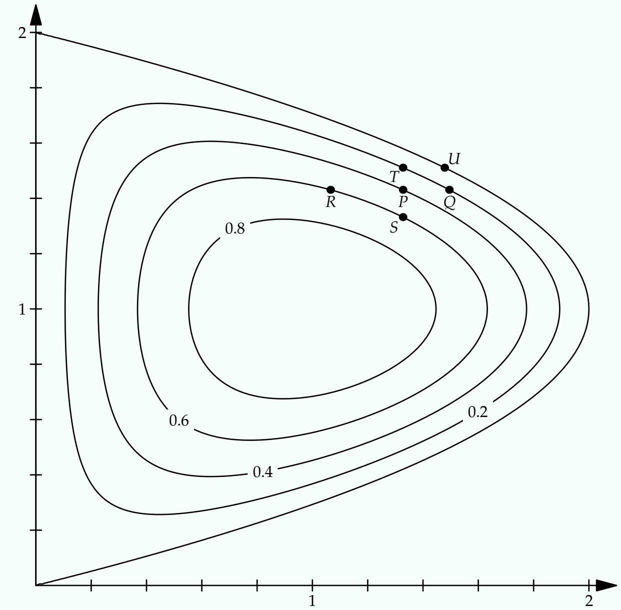
Show that $u(x, y) = e^x \sin y$ satisfies Laplace's equation.

Solution

Problem 4

A contour plot of a funtion f is shown to the right. Determine the signs of f_x, f_y, f_{xx}, f_{xy}, and f_{yy} at the point P.

The points Q, R, S, T, and U are labeled on the diagram for your convenience: R and Q are due west and east of P, respectively, while S and T are due south and north, respectively. Point U is due east of T. You will want to consider the slopes of secant lines passing through various pairs of these points.



Solution

Problem 5

Consider the equation $z = x^2 - xy + 3y^2$. As (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, find the change in the value of z. Repeat the exercise with $z = x^2 - xy + 3y^2$ replaced by the plane tangent to $z = x^2 - xy + 3y^2$ at the point $(3, -1)$.

Solution

Additional space