Brown University
Problem Set 3
Instructor: Samuel S. Watson
Due: 29 September 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

An astronaut is using a rope to move in space in such a way that her position at time t is given by $\mathbf{r}(t) = (2+t)\mathbf{i} + (2+\ln t)\mathbf{j} + \left(7 - \frac{4}{t^2+1}\right)\mathbf{k}$. The coordinates of the space station doorway are $(5, 4, 9)$. When should the astronaut let go of the rope so as to drift into the doorway?

Solution

Problem 2

Find the curvature of the helix $\mathbf{r}(t) = \langle a\cos t, a\sin t, bt\rangle$.

Solution

Problem 3

Sketch the surface or solid described by the given equations and/or inequalities. If you feel your figure isn't sufficiently clear, feel free to supplement with a verbal description.

(a) $r = 3, \quad -1 \leq z \leq 1$

(b) $\rho = 2, \quad \pi/3 \leq \phi \leq 2\pi/3$

(c) $1 \leq r \leq 3, \quad -2 \leq z \leq 2$

Solution

Problem 4

Suppose that $\mathbf{r} : [a, b] \to \mathbb{R}^3$ is a curvy path with no straight portions. Given any positive integer n and real numbers $t_0, \ldots, t_n$ satisfying

$$a = t_0 < t_1 < \cdots < t_n = b,$$

explain briefly and in simple terms why

$$|\mathbf{r}(t_1) - \mathbf{r}(t_0)| + |\mathbf{r}(t_2) - \mathbf{r}(t_1)| + \cdots + |\mathbf{r}(t_n) - \mathbf{r}(t_{n-1})|$$

is less than the length of $\mathbf{r}$. Hint: draw a figure wherein the above expression has a natural geometric interpretation.

Solution

Problem 5

(a) The set of points satisfying $z = x^2$ and $y = 0$ is revolved around the z-axis. Write an equation for the surface generated in rectangular coordinates, and in cylindrical coordinates.

(b) The set of points satisfying $4x^2 + y^2 = 1$ and $z = 0$ is revolved around the y-axis. Find an equation in rectangular coordinates for the resulting ellipsoid.

Solution

Additional space