BROWN UNIVERSITY
PROBLEM SET 2
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DUE: 22 SEPTEMBER 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

If a and b are scalars and $\mathbf{u}$ is a vector in $\mathbb{R}^3$, then $(ab)\mathbf{u} = a(b\mathbf{u})$. Explain the difference between the meaning of $(ab)\mathbf{u}$ and the meaning of $a(b\mathbf{u})$ and use coordinates to show that the two sides are in fact equal.

Solution

Problem 2

Suppose that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors such that $\mathbf{u} \cdot \mathbf{v} = 3$, $|\mathbf{u}| = 4$, $|\mathbf{w}| = 2$, and the cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$ is $\frac{3}{4}$. Show that $\mathbf{u}$ is perpendicular to $-2\mathbf{v} + \mathbf{w}$.

Solution

Problem 3

Find the determinant of each of the following matrices, and draw the image of the unit square under the corresponding linear transformations to see that value of the determinant you computed makes sense.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Solution

Problem 4

Find two vectors **u** and **v** which are both perpendicular to $\langle -1, 4, 3 \rangle$ and are perpendicular to each other.

Solution

Problem 5

Use dot products to show that the diagonals of a parallelogram have the same length if and only if the parallelogram is a rectangle. (Hints: let **a** and **b** be vectors along two sides of the parallelogram, and express vectors running along the diagonals in terms of **a** and **b**.)

Solution

Problem 6

Find the distance from the origin to the plane $x + 2y + 3z = 6$.

Solution

Problem 7

The line L_1 is described by the parametric equation $(x(t), y(t), z(t)) = (3 + 2t, t, 4)$, and the line L_2 passes through the points $P(2, 1, -3)$ and $Q(0, 8, 4)$. These two lines are *skew*, meaning that they do not intersect and are not parallel. Find the shortest possible distance between a point on L_1 and a point on L_2.

Solution

Additional space