**Brown University**
**Math 0350 Practice Midterm I**
**Instructor: Samuel S. Watson**

Name:

### Problem 1

How many hemispheres $H$ have the property that (i) $H$ is a subset of the unit sphere $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$, and (ii) $H$ is the graph of some real-valued function $f$ defined on a subset of $\mathbb{R}^2$? Explain your answer.

### Solution

Final answer:

### Problem 2

Suppose that $f(x, y) = (ax + by, cx + dy)$ has the property that $f(x, y)$ is the point obtained by rotating the point $(x, y)$ by 42 degrees counterclockwise about the origin. Find $bc - ad$.

### Solution

Final answer:

**Problem 3**

Find the distance from the plane $3x + 2y + z = 6$ to the line passing through the point $(3, 4, 5)$ and parallel to the vector $\langle -2, 3, 0 \rangle$.

**Solution**

Final answer:

**Problem 4**

The curve in $\mathbb{R}^3$ represented parametrically in $\mathbb{R}^3$ by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ is called the *twisted cubic*. Find a point on the twisted cubic at which the tangent line is parallel to the vector $\langle 4, 16, 48 \rangle$.

**Solution**

Final answer:

## Problem 5

The parallelogram law states that the sum of squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the four sides of the parallelogram. Use vectors to prove the parallelogram law. (Hint: represent the two sides as **a** and **b**, and represent the two diagonals in terms of **a** and **b**.)

## Solution

## Problem 6

Suppose that $f$ is a differentiable function from $\mathbb{R}^2$ to $\mathbb{R}$ with the property that $f_x = 1 - 4y\sin(2x)$ and $f_y = 2\cos 2x$.

(a) Explain why this is not enough information to approximate $f(0.1, 0.02)$.

(b) Approximate $f(0.1, 0.02) - f(0, 0)$ using the linear approximation of $f$.

## Solution

**Problem 7**

Your friend slices her spherical orange into 8 congruent pieces using three cuts. Write a system of inequalities in spherical coordinates whose graph is shaped like one of her pieces.

**Solution**

**Problem 8**

(a) Determine whether each statement is true or false, and explain your answer.

(i) If every "$z =$ constant" slice of a surface is a circle, then the surface must be a sphere.

(ii) The graph of $z = 2x^2 - 2y^2$ is shaped like a saddle.

(iii) If every "$x =$ constant" slice of a quadric surface is a parabola, then every "$y =$ constant" slice is a parabola.

(b) Sketch the graph of $z(z-1)(z+1) = x^2 + y^2$.

**Solution**

## Problem 9

Determine each of the following limits or explain why it doesn't exist.

(a) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y^2}{x^2+y^4}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^2+y^2}$

## Solution

**Problem 10**

Suppose that $f : \mathbb{R}^2 \to \mathbb{R}$ is a function with the property that $f\left(\frac{1}{n}, \frac{1}{\sqrt{n}}\right) = 0$ for all positive integers $n$. Explain in rigorous terms why it cannot be true that $\lim_{(x,y)\to(0,0)} f(x,y) = 1$.

**Solution**