Math 0350 Practice Final
Fall 2017
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Problem 1

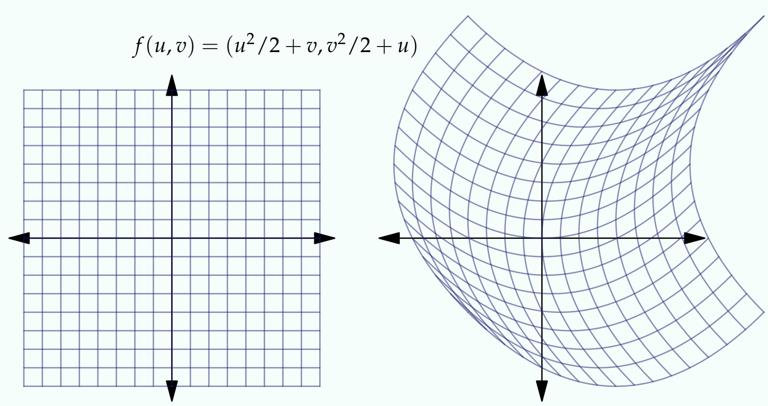
Verify that if **a** and **b** are nonzero vectors, the vector $\mathbf{c} = |\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$ bisects the angle between **a** and **b**.

Solution

Problem 2

Consider the transformation shown, which maps the square $[-1,1]^2$ to an awesome manta-ray-looking region. (a) Where are the two points in the domain of f where the Jacobian of f is equal to 0? Where is the Jacobian of f at a maximum?

(b) Does the transformation map its domain onto its image in an orientation-preserving way or an orientation-reversing way?



Solution

Problem 3

Suppose that $f(x, y) = xy - x$. Find the set of real numbers c such that there exists a differentiable path $\mathbf{r}$ satisfying $\mathbf{r}(0) = \mathbf{0}$ and $(f \circ \mathbf{r})'(0) = c$.

Solution

Problem 4

Find the set of all points on the plane $x + y + z = 1$ which are equidistant from the points $(4, 2, 2)$, and $(3, 5, 1)$.

Solution

Problem 5

Find the center of mass of the parabolic lamina $0 \leq y \leq 1 - x^2$.

Solution

Final answer:

Problem 6

Find the moment of inertia about the z-axis of the tetrahedron with vertices at the origin, $(1,0,0)$, $(1,1,0)$, and $(1,1,1)$. Assume that the mass density of the tetrahedron is $\rho(x,y,z)=1$.

Solution

Final answer:

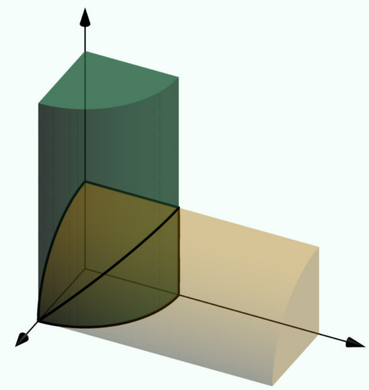
Problem 7

Find the critical point of $f(x, y) = xy + 2x - \ln(x^2y)$ in the first quadrant, and determine whether f has a local maximum, a local minimum, or a saddle point there.

Solution

Problem 8

Find the volume of the region in the first octant common to the cylinders $x^2 + y^2 \leq 1$ and $x^2 + z^2 \leq 1$, as shown.



Solution

Final answer:

Problem 9

Find the flow of $\mathbf{F} = \langle 0, 0, x^2 + y^2 \rangle$ outward through the portion of the surface $z^2 + 1 = x^2 + y^2$ between the planes $z = 0$ and $z = \sqrt{3}$.

Solution

Final answer:

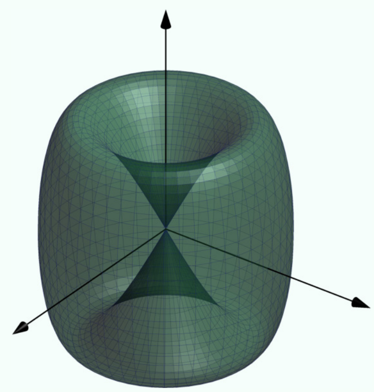
Problem 10

Consider the surface parametrized by

$$\mathbf{r}(u, v) = (\cos u \cos v, \cos u \sin v, \sin 2u).$$

as u ranges over $[-\pi/2, \pi/2]$ and v ranges over $[0, 2\pi]$. Use the divergence theorem to find the volume enclosed by the surface.

Hint: you'll have occasion to make use of the identities $\sin 2\theta = 2 \sin\theta \cos\theta$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$.



Solution

Final answer:

Problem 11

Suppose that S_1 is the set of points on the sphere $x^2 + y^2 + z^2 = 1$ which are not inside the sphere $x^2 + y^2 + (z+1)^2 = 1$, and suppose that S_2 is the set of points on the sphere $x^2 + y^2 + (z+1)^2 = 1$ which are not inside the sphere $x^2 + y^2 + z^2 = 1$. We may interpret S_1 and S_2 as surfaces carrying an orientation from the inside to the outside. Find

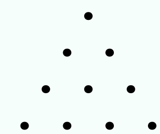
$$\iint_{S_1} \nabla \times \mathbf{F} \cdot d\mathbf{A} \quad \text{and} \quad \iint_{S_2} \nabla \times \mathbf{F} \cdot d\mathbf{A},$$

where $\mathbf{F} = \langle yz, x, e^{xyz} \rangle$.

Solution

BONUS

How many distinct vectors can be formed by selecting a tail and a head from the grid of points shown?



Solution

Final answer: